RELATIVISTIC FLUIDS AND THE PHYSICS OF GRAVITATIONAL COLLAPSE

1 Questions

- Why is the study of gravitational collapse important for General Relativity?
- Why is the gravitational collapse a highly dissipative process.
- Is there any life between quasi–equilibrium and non–equilibrium?
- What happens if we relax the Pascal principle? (and why?)
- How does the electric charge and dissipation affects the evolution of massive stars?

2 Why is the gravitational collapse a highly dissipative process?

$$E \approx -\frac{GM^2}{R} \approx -\frac{6.6 \times 10^{-8} g^{-1} cm^3 s^{-2} 10^{66} g^2}{10^6 cm} \tag{1}$$

$M = M_\odot \approx 2.10^{33} g; \quad R \approx 10 Km$

$$E \approx -10^{53} erg. \tag{2}$$

$$E_{in} \approx kT \tag{3}$$

$k \approx 1.3 \times 10^{-16} erg.K^{-1}$

$$T \approx 10^{69} K \tag{4}$$

$$L \approx \sigma T^4 R^2 \tag{5}$$

$\sigma \approx 5.6 \times 10^{-5} erg.cm^{-2} s^{-1} K^{-4}$

$$L \approx 10^{283} erg.s^{-1} \tag{6}$$

$$t \approx (10)^{-230} s \tag{7}$$

3 Non–comoving (Bondi 1964).

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right), \quad (8)$$

$$G^{\nu}_{\mu} = 8\pi T^{\nu}_{\mu}, \quad (9)$$

$$\left(\begin{array}{cccc} \rho+\epsilon & -q-\epsilon & 0 & 0 \\ -q-\epsilon & P_r+\epsilon & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\perp \end{array}\right).$$

$$\frac{\rho + P_r\omega^2}{1-\omega^2}+\frac{2\omega q}{1-\omega^2}+\frac{\epsilon(1+\omega)}{1-\omega} = -\frac{1}{8\pi}\Big\{-\frac{1}{r^2}+e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)\Big\}, \quad (10)$$

$$\frac{P_r + \rho\omega^2}{1-\omega^2}+\frac{2\omega q}{1-\omega^2}+\frac{\epsilon(1+\omega)}{1-\omega} = -\frac{1}{8\pi}\Big\{\frac{1}{r^2}-e^{-\lambda}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right)\Big\}, \quad (11)$$

$$P_\perp = -\frac{1}{8\pi}\Big\{\frac{e^{-\nu}}{4}\left(2\ddot{\lambda}+\dot{\lambda}(\dot{\lambda}-\dot{\nu})\right)$$
$$-\frac{e^{-\lambda}}{4}\left(2\nu''+\nu'^2-\lambda'\nu'+2\frac{\nu'-\lambda'}{r}\right)\Big\}, \quad (12)$$

$$\frac{(\rho+P_r)\omega e^{\frac{\lambda+\nu}{2}}}{1-\omega^2}+\frac{qe^{\frac{\lambda+\nu}{2}}}{1-\omega^2}(1+\omega^2)+\frac{e^{\frac{\lambda+\nu}{2}}\epsilon(1+\omega)}{1-\omega}=-\frac{\dot{\lambda}}{8\pi r}. \tag{13}$$

$$T^{\mu}_{\nu;\mu}=0. \tag{14}$$

$$\left(-8\pi T^1_1\right)'=\frac{16\pi}{r}\left(T^1_1-T^2_2\right)+4\pi\nu'\left(T^1_1-T^0_0\right)+\frac{e^{-\nu}}{r}\left(\ddot{\lambda}+\frac{\dot{\lambda}^2}{2}-\frac{\dot{\lambda}\dot{\nu}}{2}\right), \tag{15}$$

$$P_r'=-\frac{m+4\pi P_r r^3}{r\left(r-2m\right)}\left(\rho+P_r\right)+\frac{2\left(P_\perp-P_r\right)}{r}, \tag{16}$$

$$m(r,t)=\frac{r}{2}(1-e^{-\lambda})=4\pi\int_0^r T^0_0 r^2 dr \tag{17}$$

3.1 Equilibrium $\omega=0$, and quasi–equilibrium $\omega^2\approx 0$

$$\ddot{\lambda}+\frac{\dot{\lambda}^2}{2}-\frac{\dot{\nu}\dot{\lambda}}{2}=8\pi r e^{\nu}\left[(P_r+\epsilon)'+(\rho+P_r+2\epsilon)\frac{\nu'}{2}-2\frac{P_\perp-P_r-\epsilon}{r}\right]. \tag{18}$$

$$\ddot{\lambda}\approx\dot{\nu}\dot{\lambda}\approx\dot{\lambda}^2\approx 0, \tag{19}$$

$$\ddot{\nu} \approx 0.$$

$$\dot{\omega} \approx O(\ddot{\lambda}, \dot{\lambda}\omega, \dot{\nu}\omega). \tag{20}$$

$$O(\omega^2) = \dot{\lambda}^2 = \dot{\nu}^2 = \dot{\lambda}\dot{\nu} = \ddot{\lambda} = \ddot{\nu} = 0, \tag{21}$$

3.2 Effective variables and the post–quasi–static approximation: The life between quasi–equilibrium and non–equilibrium

$$\tilde{\rho} = T_0^0 = \frac{\rho + P_r\omega^2}{1-\omega^2} + \frac{2q\omega}{(1-\omega^2)} + \epsilon\frac{(1+\omega)}{1-\omega}, \tag{22}$$

$$\tilde{P} = -T_1^1 = \frac{P_r + \rho\omega^2}{1-\omega^2} + \frac{2q\omega}{(1-\omega^2)} + \epsilon\frac{(1+\omega)}{1-\omega}. \tag{23}$$

$$m = \frac{r}{2}(1 - e^{-\lambda}) = 4\pi\int_0^r r^2\tilde{\rho}dr, \tag{24}$$

$$\nu = \nu_\Sigma + \int_{r_\Sigma}^r \frac{2(4\pi r^3\tilde{P} + m)}{r(r-2m)}dr. \tag{25}$$

3.3 The algorithm for modelling spheres out of equilibrium

1. Take an interior (seed) solution to Einstein equations, representing a fluid distribution of matter in equilibrium, with a given

$$\rho_{st} = \rho(r) \qquad P_{r\,st} = P_r(r)$$

2. Assume that the r–dependence of $\tilde{P}$ and $\tilde{\rho}$ is the same as that of $P_{r\,st}$ and ρ_{st}, respectively.

3. Using equations (25) and (24), with the r dependence of $\tilde{P}$ and $\tilde{\rho}$, one gets m and ν up to some functions of t, which will be specified below.

4. For these functions of t one has three ordinary differential equations (hereafter referred to as surface equations), namely:

   (a) $\omega = \dot{r}_\Sigma \, e^{(\lambda-\nu)/2}$ evaluated on $r = r_\Sigma$.

   (b) $T^\mu_{r;\mu} = 0$ evaluated on $r = r_\Sigma$.

   (c) The equation relating the total mass loss rate with the energy flux through the boundary surface.

5. Depending on the kind of matter under consideration, the system of surface equations described above

may be closed with the additional information provided by the transport equation and/or the equation of state for the anisotropic pressure and/or additional information about some of the physical variables evaluated on the boundary surface (e.g. the luminosity).

6. Once the system of surface equations is closed, it may be integrated for any particular initial data.

7. Feeding back the result of integration in the expressions for m and ν, these two functions are completely determined.

8. With the input from the point 7 above, and using field equations, together with the equations of state and/or transport equation, all physical variables may be found for any piece of matter distribution.

4 Relaxing the Pascal principle? (and why?)

$$P_r' = -\frac{m + 4\pi P_r r^3}{r\,(r-2m)}\,(\rho + P_r) + \frac{2\,(P_\perp - P_r)}{r}, \qquad (26)$$

$\frac{P_r + P_\perp}{2}$

- What might be the origin of local anisotropy?

1. "Exotic" phase transition (e. g. pion condensate).
2. Magnetic fields
3. Type II superconductor
4. Type P superfluid
5. Boson stars
6. Viscosity
7. Anisotropic velocity distributions
8. Two fluid systems

- How does the properties of the locally anisotropic system differs from the locally isotropic one?

1. Cracking induced by perturbations of local isotropy of pressure. $\frac{(P_\perp - P_r)}{P_r} << 1$
2. Changes in the total mass allowed for a compact object

5 Comoving (Misner and Sharp 1964).

$$ds_-^2 = -A^2dt^2 + B^2dr^2 + (Cr)^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (27)$$

$$\begin{aligned} T^-_{\alpha\beta} &= (\mu + P_\perp)V_\alpha V_\beta + P_\perp g_{\alpha\beta} + (P_r - P_\perp)\chi_\alpha\chi_\beta \\ &+ q_\alpha V_\beta + V_\alpha q_\beta + \epsilon l_\alpha l_\beta - 2\eta\sigma_{\alpha\beta}, \end{aligned} \quad (28)$$

$$\begin{aligned} &V^\alpha V_\alpha = -1, \quad V^\alpha q_\alpha = 0, \quad \chi^\alpha\chi_\alpha = 1, \\ &\chi^\alpha V_\alpha = 0, \qquad l^\alpha V_\alpha = -1, \quad l^\alpha l_\alpha = 0, \end{aligned} \quad (29)$$

$$\sigma_{\alpha\beta} = V_{(\alpha;\beta)} + a_{(\alpha}V_{\beta)} - \frac{1}{3}\Theta(g_{\alpha\beta} + V_\alpha V_\beta), \quad (30)$$

$$a_\alpha = V_{\alpha;\beta}V^\beta, \quad \Theta = V^\alpha{}_{;\alpha}. \quad (31)$$

$$\begin{aligned} V^\alpha &= A^{-1}\delta_0^\alpha, \qquad q^\alpha = qB^{-1}\delta_1^\alpha, \\ l^\alpha = A^{-1}\delta_0^\alpha &+ B^{-1}\delta_1^\alpha, \qquad \chi^\alpha = B^{-1}\delta_1^\alpha, \end{aligned} \quad (32)$$

$$\sigma_{11} = \frac{2}{3}B^2\sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\sin^2\theta} = -\frac{1}{3}(Cr)^2\sigma, \quad (33)$$

$$\sigma = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{34}$$

$$\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{2}{3}\sigma^2. \tag{35}$$

$$a_1 = \frac{A'}{A}, \quad \Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + 2\frac{\dot{C}}{C}\right), \tag{36}$$

5.1 The electromagnetic energy tensor and the Maxwell equations

$$E^{-}_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha}{}^{\gamma}F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right), \tag{37}$$

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}, \tag{38}$$

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^{\alpha}, \tag{39}$$

$$\phi_{\alpha} = \Phi\delta^{0}_{\alpha}, \quad J^{\alpha} = \varsigma V^{\alpha}, \tag{40}$$

$$s(r) = 4\pi\int_0^r \varsigma B(Cr)^2 dr, \tag{41}$$

$$\Phi'' - \left(\frac{A'}{A} + \frac{B'}{B} - 2\frac{C'}{C} - \frac{2}{r}\right)\Phi' = 4\pi\varsigma AB^2, \tag{42}$$

$$\dot{\Phi}' - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - 2\frac{\dot{C}}{C}\right)\Phi' = 0. \tag{43}$$

$$\Phi' = \frac{sAB}{(Cr)^2}. \tag{44}$$

5.2 The Einstein equations

$$G^-_{\alpha\beta} = 8\pi(T^-_{\alpha\beta} + E^-_{\alpha\beta}). \tag{45}$$

$$\begin{aligned}
&8\pi(T^-_{00} + E^-_{00}) = 8\pi(\mu+\epsilon)A^2 + \frac{(sA)^2}{(Cr)^4} \\
&= \left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\frac{\dot{C}}{C} + \left(\frac{A}{B}\right)^2 \left\{-2\frac{C''}{C} + \left(2\frac{B'}{B} - \frac{C'}{C}\right)\frac{C'}{C}\right. \\
&\left.+\frac{2}{r}\left(\frac{B'}{B} - 3\frac{C'}{C}\right) - \left[1 - \left(\frac{B}{C}\right)^2\right]\frac{1}{r^2}\right\},
\end{aligned} \tag{46}$$

$$\begin{aligned}
&8\pi(T^-_{01} + E^-_{01}) = -8\pi(q+\epsilon)AB \\
&= -2\left(\frac{\dot{C}'}{C} - \frac{\dot{B}}{B}\frac{C'}{C} - \frac{\dot{C}}{C}\frac{A'}{A}\right) + \frac{2}{r}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right),
\end{aligned} \tag{47}$$

$$\begin{aligned}
&8\pi(T^-_{11} + E^-_{11}) = 8\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)B^2 - \frac{(sB)^2}{(Cr)^4} \\
&= -\left(\frac{B}{A}\right)^2\left[2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - 2\frac{\dot{A}}{A}\frac{\dot{C}}{C}\right] \\
&+\left(\frac{C'}{C}\right)^2 + 2\frac{A'}{A}\frac{C'}{C} + \frac{2}{r}\left(\frac{A'}{A} + \frac{C'}{C}\right) + \left[1 - \left(\frac{B}{C}\right)^2\right]\frac{1}{r^2},
\end{aligned} \tag{48}$$

$$
\begin{aligned}
8\pi(T^{-}_{22}+E^{-}_{22}) &= \frac{8\pi}{\sin^2\theta}(T^{-}_{33}+E^{-}_{33}) \\
&= 8\pi\left(P_\perp+\frac{2}{3}\eta\sigma\right)(Cr)^2+\left(\frac{s}{Cr}\right)^2 \\
&= -\left(\frac{Cr}{A}\right)^2\left[\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}-\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{\dot{B}}{B}\frac{\dot{C}}{C}\right] \\
&+\left(\frac{Cr}{B}\right)^2\left[\frac{A''}{A}+\frac{C''}{C}-\frac{A'}{A}\left(\frac{B'}{B}-\frac{C'}{C}\right)-\frac{B'}{B}\frac{C'}{C}\right. \\
&\left.+\frac{1}{r}\left(\frac{A'}{A}-\frac{B'}{B}+2\frac{C'}{C}\right)\right]. \qquad (49)
\end{aligned}
$$

$$
m=\frac{(Cr)^3}{2}R_{23}{}^{23}+\frac{s^2}{2Cr}=\frac{Cr}{2}\left\{\left(\frac{r\dot{C}}{A}\right)^2-\left[\frac{(Cr)'}{B}\right]^2+1\right\}+\frac{s^2}{2Cr}, \qquad (50)
$$

5.3 The exterior spacetime and junction conditions

$$
ds^2=-\left[1-\frac{2M(v)}{r}+\frac{Q^2}{r^2}\right]dv^2-2drdv+r^2(d\theta^2+r^2\sin\theta^2d\phi^2) \qquad (51)
$$

$$
P_r-\frac{4\eta\sigma}{3}\overset{\Sigma}{=}q, \quad m(t,r)\overset{\Sigma}{=}M(v), \quad s\overset{\Sigma}{=}Q, \qquad (52)
$$

6 Dynamical equations

The non trivial components of the Bianchi identities, $(T^{-\alpha\beta}+E^{-\alpha\beta})_{;\beta}=0$,

$$\begin{aligned}(T^{-\alpha\beta}+E^{-\alpha\beta})_{;\beta}V_{\alpha} &= -\frac{1}{A}(\dot{\mu}+\dot{\epsilon})-\left(\mu+P_r+2\epsilon-\frac{4}{3}\eta\sigma\right)\frac{\dot{B}}{AB}\\ &-2\left(\mu+P_{\perp}+\epsilon+\frac{2}{3}\eta\sigma\right)\frac{\dot{C}}{AC}\\ &-\frac{1}{B}(q+\epsilon)'-2(q+\epsilon)\frac{(ACr)'}{ABCr} = 0,\end{aligned} \tag{53}$$

$$\begin{aligned}(T^{-\alpha\beta}+E^{-\alpha\beta})_{;\beta}\chi_{\alpha} &= \frac{1}{A}(\dot{q}+\dot{\epsilon})+\frac{1}{B}\left(P_r+\epsilon-\frac{4}{3}\eta\sigma\right)'\\ &+2(q+\epsilon)\frac{\dot{B}}{AB}+2(q+\epsilon)\frac{\dot{C}}{AC}\\ &+\left(\mu+P_r+2\epsilon-\frac{4}{3}\eta\sigma\right)\frac{A'}{AB}\\ &+2(P_r-P_{\perp}+\epsilon-2\eta\sigma)\frac{(Cr)'}{BCr}-\frac{ss'}{4\pi B(Cr)^4} = 0,\end{aligned} \tag{54}$$

$$D_T=\frac{1}{A}\frac{\partial}{\partial t}, \tag{55}$$

$$D_R=\frac{1}{R'}\frac{\partial}{\partial r}, \tag{56}$$

$$R=Cr, \tag{57}$$

$$U = rD_T C = D_T R < 0 \ \text{(in the case of collapse)}. \tag{58}$$

$$E \equiv \frac{(Cr)'}{B} = \left[1 + U^2 - \frac{2m(t,r)}{Cr} + \left(\frac{s}{Cr}\right)^2\right]^{1/2}. \tag{59}$$

$$D_T m = -4\pi \left[\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right) U + (q+\epsilon)E\right] R^2, \tag{60}$$

$$D_R m = 4\pi \left[\mu + \epsilon + (q+\epsilon)\frac{U}{E}\right] R^2 + \frac{s}{R} D_R s. \tag{61}$$

$$m = \int_0^R 4\pi R^2 \left[\mu + \epsilon + (q+\epsilon)\frac{U}{E}\right] dR + \frac{s^2}{2R} + \frac{1}{2}\int_0^R \frac{s^2}{R^2} dR \tag{62}$$

(assuming a regular centre to the distribution, so $m(0) = 0$).

$$D_T U = -\frac{m}{R^2} - 4\pi \left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right) R + \frac{s^2}{R^3} + \frac{EA'}{AB}, \tag{63}$$

$$
\begin{aligned}
&\left(\mu + P_r + 2\epsilon - \frac{4}{3}\eta\sigma\right) D_T U = \\
&-\left(\mu + P_r + 2\epsilon - \frac{4}{3}\eta\sigma\right)\left[m + 4\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)R^3 - \frac{s^2}{R}\right]\frac{1}{R^2} \\
&-E^2\left[D_R\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right) + 2(P_r - P_\perp + \epsilon - 2\eta\sigma)\frac{1}{R}\right. \\
&\left.-\frac{s}{4\pi R^4}D_R s\right] - E\left[D_T q + D_T\epsilon + 4(q+\epsilon)\frac{U}{R} + 2(q+\epsilon)\sigma\right],
\end{aligned} \tag{64}
$$

$$
Force = Mass\ (density) \times Acceleration \tag{65}
$$

$$
\begin{aligned}
&\int_0^R 4\pi R^2\left[\mu + \epsilon + (q+\epsilon)\frac{U}{E}\right]dR + 4\pi(P_r + \epsilon - \frac{4}{3}\eta\sigma)R^3 \ - \\
&-\frac{s^2}{2R} + \frac{1}{2}\int_0^R \frac{s^2}{R^2}dR
\end{aligned} \tag{66}
$$

$$
\int_0^R \frac{s^2}{R^2}dR > \frac{s^2}{R} \ \ \text{(increases active grav. mass term)} \tag{67}
$$

$$
\int_0^R \frac{s^2}{R^2}dR < \frac{s^2}{R} \ \ \text{(decreases active grav. mass term)} \tag{68}
$$

$$
\int_0^R \frac{s^2}{R^2}dR = \frac{s^2}{R} \ \ \text{(no regenerative effect of charge)} \tag{69}
$$

$$
s \sim R \tag{70}
$$

7 The transport equation

7.1 Maxwell-Fourier law and causality

$$\vec{q}(\vec{x},t) \;=\; -\kappa\vec{\nabla}T(\vec{x},t) \tag{71}$$

$$\mathrm{d}e = \gamma \mathrm{d}T \qquad \text{and} \qquad \frac{\mathrm{d}e}{\mathrm{d}t} = -\vec{\nabla}\cdot\vec{q} \tag{72}$$

$$\boxed{\frac{\partial T}{\partial t} = \frac{\kappa}{\gamma}\nabla^2 T} \tag{73}$$

$$T \sim \frac{1}{\sqrt{t}}\exp\left[-\frac{(x-x_0)^2}{t}\right] \tag{74}$$

$$t = 0 : \; T = \delta(x - x_0)$$
$$t = \tilde{t} > 0 : \; T \neq 0$$

$$\vec{q}(\vec{x}, t+\tau) = -\kappa \vec{\nabla} T(\vec{x}, t) \tag{75}$$

$$\boxed{\tau \frac{\partial \vec{q}}{\partial t} + \vec{q} = -\kappa \vec{\nabla} T} \qquad \text{Cattaneo} \tag{76}$$

$$\boxed{\frac{\kappa}{\tau\gamma} \nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau} \frac{\partial T}{\partial t}} \tag{77}$$

$$c = \sqrt{\frac{\kappa}{\tau\gamma}} \tag{78}$$

$$\vec{q}(\vec{x}, t) = -\frac{\kappa}{\tau} \int_{-\infty}^{t} \exp\left[-\frac{(t-t')}{\tau}\right] \cdot \vec{\nabla} T(\vec{x}, t') dt' \tag{79}$$

$$\vec{q}(\vec{x}, t) = -\int_{-\infty}^{t} Q(t-t') \vec{\nabla} T(\vec{x}, t') dt' \tag{80}$$

$$\begin{array}{llll} \text{if} \;\; Q = K\delta(t-t') & \Longrightarrow & \vec{q} = -\kappa \vec{\nabla} T & \text{(Fourier)} \\ \text{if} \;\; Q = \text{constant} & \Longrightarrow & \frac{\partial^2 T}{\partial t^2} = \frac{\kappa}{\gamma} \nabla^2 T & \end{array} \tag{81}$$

$$\left\{ \begin{array}{lcl} \tau & \approx & 10^{-11}\text{sec.} & (\text{phonon} - \text{elect.}) \\ \tau & \approx & 10^{-13}\text{sec.} & (\text{phonon} - \text{phonon}) \\ \tau & \approx & 10^{-3}\text{sec.} & HeII \simeq 1.2^{\circ}K \\ \tau & \approx & 10^{-1} - 10^{-4}\text{sec.} & \text{neutron} - \text{star} - \text{matter} \end{array} \right\} \tag{82}$$

$$\tau h^{\alpha\beta} V^{\gamma} q_{\beta;\gamma} + q^{\alpha} = -\kappa h^{\alpha\beta}(T_{,\beta} + T a_{\beta}) - \frac{1}{2}\kappa T^2 \left(\frac{\tau V^{\beta}}{\kappa T^2}\right)_{;\beta} q^{\alpha}, \tag{83}$$

$$\begin{aligned} D_T q &= -\frac{\kappa T^2 q}{2\tau} D_T \left(\frac{\tau}{\kappa T^2}\right) - q\left(\frac{3}{2}\frac{U}{R} + \frac{1}{2}\sigma + \frac{1}{\tau}\right) \\ &\quad -\frac{\kappa E}{\tau} D_R T - \frac{\kappa T}{\tau E} D_T U \\ &\quad -\frac{\kappa T}{\tau E}\left[m + 4\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right) R^3 - \frac{s^2}{R}\right]\frac{1}{R^2}. \end{aligned} \tag{84}$$

$$
\begin{aligned}
& \left(\mu + P_r + 2\epsilon - \frac{4}{3}\sigma\eta\right)(1-\alpha) D_T U \\
= \ & (1-\alpha) F_{grav} + F_{hyd} + \frac{\kappa E^2}{\tau} D_R T \\
+ \ & E\left[\frac{\kappa T^2 q}{2\tau} D_T\left(\frac{\tau}{\kappa T^2}\right) - D_T \epsilon\right] \\
- \ & Eq\left(\frac{5}{2}\frac{U}{R} + \frac{3}{2}\sigma - \frac{1}{\tau}\right) - 2E\epsilon\left(2\frac{U}{R} + \sigma\right), \quad (85)
\end{aligned}
$$

$$
\begin{aligned}
F_{grav} \quad & = -\left(\mu + P_r + 2\epsilon - \frac{4}{3}\eta\sigma\right) \\
\times \ & \left[m + 4\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right) R^3 - \frac{s^2}{R}\right]\frac{1}{R^2}, (86) \\
F_{hyd} \ = \ & -E^2\left[D_R\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)\right. \\
+ \ & \left. 2(P_r - P_\perp + \epsilon - 2\eta\sigma)\frac{1}{R} - \frac{s}{4\pi R^4} D_R s\right], \\
& (87)
\end{aligned}
$$

$$
\alpha = \frac{\kappa T}{\tau}\left(\mu + P_r + 2\epsilon - \frac{4}{3}\sigma\eta\right)^{-1}. \quad (88)
$$

$$
\alpha \approx \frac{[\kappa][T]}{[\tau]([\mu] + [P_r] + 2[\epsilon] - \frac{4}{3}[\eta\sigma])} \times 10^{-42} \quad (89)
$$

$[\kappa]$, $[T]$, $[\tau]$, $[\mu]$ en $erg.\, s^{-1}\, cm^{-1}\, K^{-1}$, K, s $g.\, cm^{-3}$.

1. Pre-supernovae
   (Martínez 96) ($[\kappa] \approx 10^{37}$; $[T] \approx 10^{13}$; $[\tau] \approx 10^{-4}$; $[\mu] \approx 10^{12}$) $\alpha \approx 1$.
2. Inflation $\mu + P = 0 \Longleftrightarrow \alpha = 1$

- Once the transport equation has been taken into account, then the inertial energy density and the "passive gravitational mass density", i.e the factor multiplying $D_T U$ and the first factor at the right of (64) respectively (which of course are the same, as expected from the equivalence principle), appear diminished by the factor $1 - \alpha$, a result already obtained, but here generalized by the inclusion of the viscosity and radiative phenomena.

- As far as the right hand side of (85) is negative, the system keeps collapsing. However, let us assume that the collapsing sphere evolves in such a way that, for some region of the sphere, the value of α increases and approaches the critical value of 1. Then, as this process goes on, the ensuing decreasing of the gravitational force term would eventually lead to a change of the sign of the right hand side of (85). Since that would happen for small values of the effective inertial mass density, that would imply a strong bouncing of that part of the sphere, even for a small absolute value of the right hand side of (85).

- Observe that the charge does not enter into the definition of α. However it does affect the "active gravi-

tational mass" (the factor within the square bracket in (86)).

- The repulsive Coulomb term (the last term in (87)) depends on $D_R s$ and always opposes gravitation. Its effect is reinforced if $D_R s$ is large enough to violate (67), in which case the charge will decrease the "active gravitational mass" term in (86).

8 Conclusions

- Dissipative phenomena may play a relevant role in the dynamics of collapse. In particular, relaxational effects may drastically change the outcome of gravitational collapse.
- The dynamical regime may be approached by means of succesive approximations.
- Local anisotropy of pressure has to be taken into consideration in the study of the structure and evolution of massive stars.
- Not only electric charge but also its distribution may be relevant in stellar structure and evolution.
- Local (non–gravitational) effects may be crucial to determine the outcome of gravitational collapse.
- Learn to deal with comoving and non–comoving frames!